Seat No. _________________ Enrollment No._________________

C. U. SHAH UNIVERSITY

M.Sc. (Mathematics) Semester-IV Summer – 2015 Regular Examination

Subject Name: Advance Graph Theory Subject Code: 5SC04AGE1

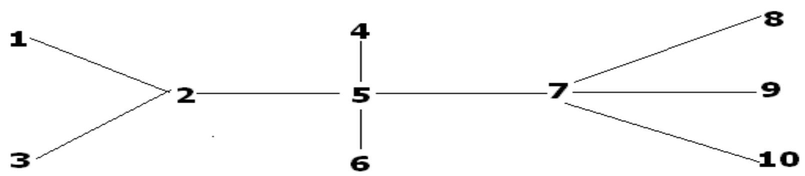
Time: 03 hours **Maximum Marks: 70**

Instructions:
1. Attempt all questions.
2. Make suitable assumption whenever necessary.
3. Figures to the right indicate full marks.

Section– I Marks

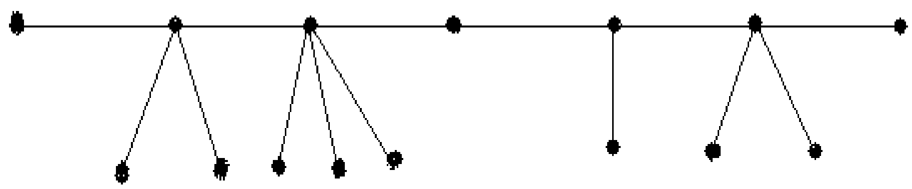
Q-1 a) Write down algorithm for pruffer code. (02)
b) Define : Network . (02)
c) Give only the statement of first theorem of graph theory. (02)
d) Define : Spanning tree. (01)

Q-2 a) For the given tree graph T find out pruffer code. (06)



b) Prove that minimum height of a binary tree with n vertices is $\lceil \log_2(n+1) - 1 \rceil$ and maximum height is $\frac{n-1}{2}$. (05)

c) Find out graceful labeling of following tree graph. (03)



OR

Q-2 a) Pruffer code of a tree T is a = (2,3,1,1,2,7) Draw the tree graph. (06)
b) Prove that number of pendant vertices in a binary tree with n vertices is $\frac{n+1}{2}$. (05)
c) Find out maximum and minimum possible height of a binary tree with 15 vertices. (03)

Q-3 a) State and prove Cayley's theorem for to find number of spanning trees for a complete graph. (07)

b) Using Matrix Tree computation method find out number of spanning trees of graphs (1) Complete bipartite graph $K_{2,3}$ and (2) Cycle C_4. (07)

OR

Q-3 a) State and prove Matrix – Tree theorem. (08)

b) Define Edge contraction . Let $\tau(G)$ denote number of spanning trees of a graph G. If $e \in E(G)$ is not a loop then prove $\tau(G) = \tau(G-e) + \tau(G.e)$ (06)

Section – II

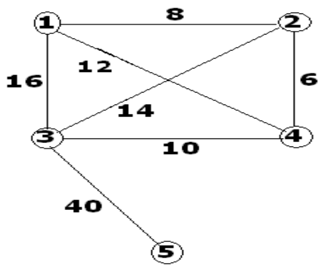
Q-4 a) Define : Matroid. (02)

b) Define : Flow augumenting path. (02)

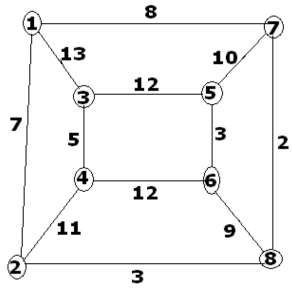
c) Explain vertex condition and edge condition in a network. (02)

d) Define : Minimum polynomial. (01)

Q-5 a) Applying Dijkstra's algorithm find out shortest path from vertex 1 to every other vertex. (06)



b) Using Kruskal's algorithm find out shortest (minimum) spanning tree in following graph G. (04)



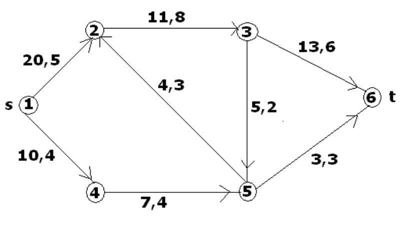
c) Explain Moore's BFS algorithm to find shortest path. (04)

OR

Q-5 a) Explain Dijkstra's algorithm. (06)

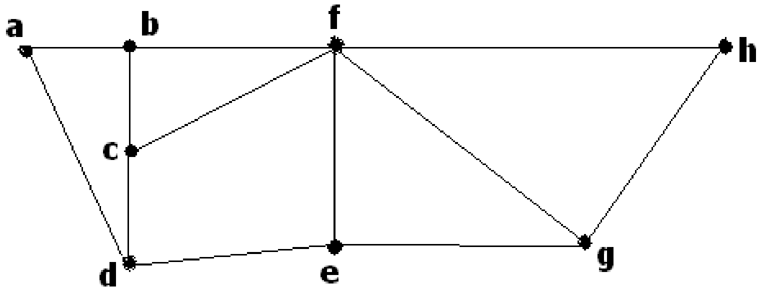
b) Explain Kruskal's algorithm. (04)

c) Find out flow augumenting paths in following network and hence find maximum possible flow. (04)



Q-6 a) Find out Eigen values of cycle graph C_4. (05)

b) Find out eccentricity of each vertex of following graph. (05)



c) State and prove Cayley – Hamilton theorem for a graph. (04)

OR

Q-6 a) Find out spectrum of complete bipartite graph $K_{2,2}$. (05)

b) If T is a spanning tree of a k-dimensional cube graph Q_k, then prove that there is an edge of Q_k outside T whose addition to T creates a cycle of length at least 2k. (05)

c) Prove that Among six persons it is possible to find out three mutual acquaintances or three mutual in acquaintances. (04)

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